Multiple object tracking
Different challenges

• Multiple objects of the same type
• Heavy occlusions
• Appearance is often very similar
Tracking-by-detection

- We will focus on algorithms where a set of detections is provided
  - Remember detections are not prefect!

Find detections that match and form a trajectory
Online vs offline tracking

- **Online tracking**
  - Often processes two frames at a time
  - For real-time applications
  - Prone to drifting → hard to recover from errors or occlusions

- **Offline tracking**
  - Processes a batch of frames
  - Good to recover from occlusions (short ones as we will see)
  - Not suitable for real-time applications
  - Suitable for video analysis
Frame-by-frame

1. Track initialization (e.g. using a detector)
Frame-by-frame

1. Track initialization (e.g. using a detector)
2. Matching detections at \( t \) with detections at \( t+1 \)
Frame-by-frame

1. Track initialization (e.g. using a detector)

2. Matching detections at $t$ with detections at $t+1$

Repeat for every pair of frames
Frame-by-frame

3. Matching tracks at $t$ with detections at $t+1$
Frame-by-frame

- Bipartite matching
  - Define distances between boxes (e.g., IoU, pixel distance, 3D distance)

<table>
<thead>
<tr>
<th>Detections</th>
<th>Detections</th>
<th>Detections</th>
<th>Detections</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.8</td>
<td>0.8</td>
<td>0.1</td>
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<tr>
<td>0.5</td>
<td>0.4</td>
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<td>0.8</td>
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<tr>
<td>0.2</td>
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<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Frame-by-frame

- Bipartite matching
  - Define distances between boxes (e.g., IoU, pixel distance, 3D distance)
  - Solve the unique matching with e.g., the Hungarian algorithm*

Frame-by-frame

- Bipartite matching
  - Define distances between boxes (e.g., IoU, pixel distance, 3D distance)
  - Solve the unique matching with e.g., the Hungarian algorithm*
  - Solutions are the unique assignments that minimize the total cost

\[
\begin{array}{cccc}
0.9 & 0.8 & 0.8 & 0.1 \\
0.5 & 0.4 & 0.3 & 0.8 \\
0.2 & 0.1 & 0.4 & 0.8 \\
0.1 & 0.2 & 0.5 & 0.9 \\
\end{array}
\]

*Demo: http://www.hungarianalgorithm.com/solve.php
Frame-by-frame

• Problems with frame-by-frame tracking
  – Cannot recover from errors. If a detection is missing from a frame, we have to end the trajectory.
  – All decisions are essentially local
  – Hard to recover from errors in the detection step

• Solution: find the minimum cost solution for ALL frames and ALL trajectories
Graph-based MOT
Tracking with network flows

Graphical model

Node
Tracking with network flows

L. Leal-Taixé et al. “Everybody needs somebody: Modeling social and grouping behavior on a linear programming multiple people tracker.” ICCVW2011
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Tracking with network flows

- Node = detection
- Edge = flow = trajectory
- 1 unit of flow = 1 pedestrian

L. Leal-Taixé et al. “Everybody needs somebody: Modeling social and grouping behavior on a linear programming multiple people tracker.” ICCVW2011
Tracking with network flows

• Solving the Minimum Cost Flow Problem

“Determine the minimum cost of shipment of a commodity through a network”

Tracking with network flows

- Solving the Minimum Cost Flow Problem

“Determine the minimum cost of shipment of a commodity through a network”

Tracking with network flows

- Objective function

\[ T^* = \arg \min_T \sum_{i,j} C(i, j) f(i, j) \]
Tracking with network flows

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\[ \mathcal{T}^* = \arg \min_{\mathcal{T}} \sum_{i,j} C(i,j) f(i,j) \]
Tracking with network flows

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Optimal set of trajectories

Indicator \{0, 1\}

Costs – what will drive the tracking
Tracking with network flows

Transition: \( \text{cost} \propto \text{distance between detections} \)

FLOW = TRAJECTORY = PEDESTRIAN
Tracking with network flows

Entrance/exit: cost to start or end a trajectory
Transition: cost $\propto$ distance between detections

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Tracking with network flows

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Flow can only start at the source node and end at the sink node
Tracking with network flows

Entrance/exit: cost to start or end a trajectory

Transition: cost $\propto$ distance between detections

What happens if all costs are positive? 

$$\mathcal{T}^* = \arg\min_{\mathcal{T}} \sum_{i,j} C(i,j)f(i,j)$$
Tracking with network flows

Entrance/exit: cost to start or end a trajectory

Transition: cost \( \propto \) distance between detections

What happens if all costs are positive? \( \mathcal{T}^* = \arg\min_{\mathcal{T}} \sum_{i,j} C(i, j)f(i, j) \)

Trivial solution: zero flow!
Tracking with network flows

- We need a negative cost

$C_{\text{det}} = \log \frac{\beta_i}{1-\beta_i}$

Complete graph
Connections that allow us to start a trajectory.
Complete graph

Connections that allow us to end a trajectory
Linear Program
MOT formulation
Why a linear program?

• Fast solvers (e.g., Simplex algorithm)

• Guaranteed to converge to the global optimum
Minimum cost flow problem

- Objective function

\[ \mathcal{T}^* = \arg \min_{\mathcal{T}} \sum_{i,j} C(i, j)f(i, j) \]
Constraints

• Objective function

\[ \mathcal{T}^* = \arg \min_{\mathcal{T}} \sum_{i,j} C(i,j) f(i,j) \]

• Subject to

Flow conservation at the nodes

\[ f_{\text{in}}(i) + f_{\text{det}}(i) = \sum_j f_t(i,j) \]

\[ \sum_j f_t(j,i) = f_{\text{out}}(i) + f_{\text{det}}(i) \]
Constraints

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Edge capacities

\[ f_{\text{in}}(i) + f_{\text{det}}(i) \in \{0, 1\} \quad f_{\text{out}}(i) + f_{\text{det}}(i) \in \{0, 1\} \quad f \in \{0, 1\} \]

NP-hard!!
LP relaxation

• Objective function

$$\mathcal{T}^* = \arg \min_{\mathcal{T}} \sum_{i,j} C(i, j) f(i, j)$$

• Subject to

Flow conservation at the nodes

$$f_{\text{in}}(i) + f_{\text{det}}(i) = \sum_j f_t(i, j) \quad \sum_j f_t(j, i) = f_{\text{out}}(i) + f_{\text{det}}(i)$$

Edge capacities

$$0 \leq f_{\text{in}}(i) + f_{\text{det}}(i) \leq 1 \quad 0 \leq f_{\text{out}}(i) + f_{\text{det}}(i) \leq 1 \quad 0 \leq f \leq 1$$
Solver

• Objective function

\[ T^* = \arg \min_T \sum_{i,j} C(i, j)f(i, j) \]

Given the shape of the constraints (total unimodularity), we solve the relaxed problem and still get integer solutions.
Objective function

• Objective function

\[ \mathcal{T}^* = \arg \min_{\mathcal{T}} \sum_i C_{\text{in}}(i)f_{\text{in}}(i) + \sum_{i,j} C_t(i,j)f_t(i,j) \]

\[ + \sum_i C_{\text{det}}(i)f_{\text{det}}(i) + \sum_i C_{\text{out}}(i)f_{\text{out}}(i) \]

\[ C(i) = -\log P(i) \]

• Equivalent to Maximum a-posteriori tracking formulation

\[ \mathcal{T}^* = \arg \max_{\mathcal{T}} \prod_j P(o_j | \mathcal{T}) P(\mathcal{T}) \]
Two ways forward

• 1. Improving the costs (aka more learning)
Two ways forward

• 2. Making the graph more complex (including more connections)
  – M. Keuper et al. „Motion segmentation and multiple object tracking by correlation co-clustering“. PAMI 2018.
  – S. Tang et al. „Multiple people tracking by lifted multicut and person reidentification“. CVPR 2017
End-to-end learning?

• Can we learn:
  – Features for multi-object tracking (e.g., costs)
  – To do data association, i.e., find a solution on the graph

• We will exploit the graph structure we have just seen and perform end-to-end learning
Message Passing Networks
General Idea

Graph with optional node and edge feature vectors

Information propagation across the graph for several iterations

Graph with updated context-aware node and (possibly edge) feature vector(s)

Figure credit: https://tkipf.github.io/graph-convolutional-networks/
General Idea

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Figure credit: https://tkipf.github.io/graph-convolutional-networks/
Learning to propagate information

- We can divide the propagation process in two steps: ‘node to edge’ and ‘edge to node’ updates.

Repeat these two updates for a fixed number of iterations (message passing steps) in order to encode context into embeddings.
'Node to edge' updates

- Notation:
  - Graph: $G = (V, E)$
  - Initial embeddings: $h_{(i,j)}^{(0)}, (i, j) \in E$, $h_i^{(0)}, i \in V$
  - Embeddings after $l$ steps: $h_{(i,j)}^{(l)}, (i, j) \in E$, $h_i^{(l)}, i \in V$
'Node to edge' updates

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  - Embeddings after $l$ steps: $h_{(i,j)}^{(l)}, (i, j) \in E$; $h_i^{(l)}, i \in V$

- At every message passing step $l$, first do:

$$h_{(i,j)}^{(l)} = \mathcal{N}_e \left( \left[ h_{i}^{(l-1)}, h_{(i,j)}^{(l-1)}, h_{j}^{(l-1)} \right] \right)$$

- Embedding of node $i$ in the precious message passing step
- Embedding of edge $(i,j)$ in the precious message passing step
- Embedding of node $j$ in the precious message passing step
‘Node to edge’ updates

- Notation:
  - Graph: \( G = (V, E) \)
  - Initial embeddings: \( h_{i,j}^{(0)}, (i, j) \in E \), \( h_i^{(0)}, i \in V \)
  - Embeddings after \( l \) steps: \( h_{i,j}^{(l)}, (i, j) \in E \), \( h_i^{(l)}, i \in V \)
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\[
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\]

Combine node and edge embeddings
'Node to edge' updates

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Learnable function (e.g. MLP) with shared weights across the entire graph

Combine node and edge embeddings
‘Edge to node’ updates

- After a round of edge updates, each edge embedding contains information about its pair of incident nodes.
‘Edge to node’ updates

- After a round of edge updates, each edge embedding contains information about its pair of incident nodes.
- Then, edge embeddings are used to update nodes:

\[ m_{(i,j)}^{(l)} = \mathcal{N}_v \left([h_i^{(l-1)}, h_{(i,j)}^{(l)}]\right) \]

Learnable function (e.g. MLP) with shared weights across the entire graph.
‘Edge to node’ updates

After a round of edge updates, each edge embedding contains information about its pair of incident nodes.

Then, edge embeddings are used to update nodes:

\[
m^{(l)}_{(i,j)} = \mathcal{N}_v \left([h^{(l-1)}_i, h^{(l)}_{(i,j)}]\right)
\]

\[
h^{(l)}_i = \Phi \left(\{m^{(l)}_{(i,j)}\}_{j \in \mathcal{N}_i}\right)
\]

Permutation invariant operation (e.g. sum, mean, max)

Neighbors of node i

The aggregation provides each node embedding with contextual information about its neighbors.
Remarks

• **Main goal**: obtaining node and edge embeddings that contain *context information* encoding graph topology and neighbor’s feature information.

• After repeating the node and edge updates for $l$ steps, each node (resp. edge) embedding contains information about all nodes (resp. edge) at distance $l$ (resp. $l - 1$) → Think of iterations as layers in a CNN

• Observe that all operations used are differentiable, hence, MPNs can be used within end-to-end pipelines

• There is vast literature on different instantiations, as well as variations of the MPN framework we presented. See Battaglia et al. for an extensive review.
MOT with Message Passing Networks
Overview
Overview

Encode appearance and scene geometry cues into node and edge embeddings.

(a) Input
(b) Graph Construction + Feature Encoding
(c) Neural Message Passing
(d) Edge Classification
(e) Output

Overview

Propagate cues across the entire graph with neural message passing

Overview

Learn to directly predict solutions of the Min-Cost Flow problem by classifying edge embeddings.

(a) Input
(b) Graph Construction + Feature Encoding
(c) Neural Message Passing
(d) Edge Classification
(e) Output

Overview

Feature Extraction

Learnable Data Association

End-to-end learning

Feature encoding

- Appearance and geometry encodings

CNN

MLP

Node embeddings

Edge embeddings

Node

Node

Appearance

Geometry

CNN

Appearance

Node embeddings

Edge embeddings
Feature encoding

- Appearance and geometry encodings

\[ \left( \frac{2(x_j - x_i)}{h_i + h_j}, \frac{2(y_j - y_i)}{h_i + h_j}, \log \frac{h_i}{h_j}, \log \frac{w_i}{w_j}, t_j - t_i \right) \]
Feature encoding

• Appearance and geometry encodings

Shared weights for all nodes and edges

Node embeddings
Edge embeddings

CNN
MLP
CNN

Appearance
Geometry
Feature encoding

• Instead of defining pairwise costs for edges and unary costs for nodes (classical setting), feature vectors encoding appearance and geometry cues are used

• **Goal:** propagate these embeddings across the entire graph in order to obtain new embeddings encoding high-order information among detections
Time-aware Message Passing

(b) Vanilla node update

(c) Time-aware node update

- All node embeddings are aggregated at once
- Aggregation of nodes is separated between past / future frames

An additional network combines both sources of aggregated features
Classifying edges

• After several iterations of message passing, each edge embedding contains high-order information about other detections

• We feed the embeddings to an MLP that predicts whether an edge is active/inactive

\[
L = \frac{-1}{|E|} \sum_{l=l_0}^{L} \sum_{(i,j) \in E} w \cdot y_{(i,j)} \log(\hat{y}_{(i,j)}^{(l)}) + (1 - y_{(i,j)}) \log(1 - \hat{y}_{(i,j)}^{(l)})
\]

- Weight to balance active / inactive edges
- Sum over the last steps
- Edge predictions (w. sigmoid) at iteration l
- Binary cross-entropy
Some results

• In practice, around 99% of constraints are automatically satisfied, and rounding takes negligible time.

• The overall method is fast (~12 fps) and achieves SOTA in the MOT Challenge by a significant margin.
MOT evaluation
Evaluation metrics

• Compute a set of measures per frame
  – Perform matching between predictions and ground truth (we will use exactly the same Hungarian algorithm)
  – FP = False positives
  – FN = False negatives (missing detections)
  – IDsw: identity switches
Evaluation metrics

• How do we compute ID switches?

(a) An ID switch is counted because the ground truth track is assigned first to red, then to blue.

(b) You count both an ID switch (red and blue both assigned to the same ground truth), but also a fragmentation (Frag) because the ground truth coverage was cut.

(c) Identity is preserved. If two trajectories overlap with a ground truth trajectory (within a threshold), the one that forces least ID switches is chosen (the red one).
Evaluation metrics

• Compute a set of measures per frame
  – Perform matching between predictions and ground truth (we will use exactly the same Hungarian algorithm)
  – FP = False positives
  – FN = False negatives (missing detections)
  – IDsw: identity switches

\[
MOTA = 1 - \frac{\sum_t (FN_t + FP_t + IDSW_t)}{\sum_t GT_t}
\]
Datasets

• MOTChallenge: www.motchallenge.net (people)
  – Several challenges from less to more crowded

• KITTI benchmark: http://www.cvlibs.net/datasets/kitti/ (vehicles)
• UA-Detrac: http://detrac-db.rit.albany.edu (vehicles)